Optimal Museum Traversal Using Graph Theory

Mitchell Eithun
Michigan State University
eithunmi@msu.edu

August 8, 2018



The Egyptian Room at the British Museum, 1844 [1].

1 Introduction

With dozens of rooms and thousands of objects on display, visiting a large museum can be overwhelming. Conscientious visitors may like to maximize their time by viewing exhibits and objects that are the most important to them, personally or intellectually, but due to the sprawling size of some museums it can be difficult to plan efficient routes that visit points of interest without visiting rooms more than once. This paper illustrates how *graph theory*, a subfield of mathematics, is helpful for formulating optimal ways to move through museums and how well museums facilitate this movement.

In graph theory, a *graph* is a collection of objects and connections between them. Graphs are useful for representing the structure or topology of something without thinking about underlying physical geometry. In this work we use graphs to summarize the layout of rooms in a museum and doorways between them. We focus on finding *Hamiltonian walks* on graphs, which translate to routes through museums that visit every room with a minimal number of repeat visits to rooms. This represents the "optimal" way to traverse a museum. We

also discuss measures of *graph connectivity* in a museum context, which describe the ease of moving between arbitrary pairs of rooms.

1.1 Museum Interaction

The field of visitor studies, largely relying on methods in sociology, focuses on how people use and interact with museums [2]. For example, in an article about museum interaction, the British Museum identified four ways that visitors view their institution: as a church, spa, archive and attraction. Thinking about the museum in these ways meets needs that are spiritual, emotional, intellectual and social, respectively [3]. Recently museums have also exploited technology such as smartphones and eye trackers to discover what people look at in museums in order to devise better visitor profiles [4]. By better understanding the habits and motivations of museums visitors, curators can create more effective exhibits.

This paper deals more with logistical questions than philosophy by discussing how museums could be explored, not necessarily how people do so in practice. Our supposed existence of an "optimal way" to move through a museum relies on the assumption that the visitor values efficiency and has an interest in the subject matter. Hence, our results are most applicable for intellectually-minded visitors identified in the British Museum's model. Evidence of associations between visiting cultural institutions and good health, including emotional well-being (see [5]), also makes a case that paths through museums could make the visiting an museum more pleasant for visitors striving to meet spiritual, emotional and social needs.

1.2 Graph Theory and Architecture

Graph theory has found several applications in architecture. The author of [6], describes how graph theory can help designers know all possible room layouts for a building, moving the discussion from intuition to enumeration. They start by modeling a floor plan as a "planar map". By finding the "dual graph", this graph can be converted into what we call the "museum graph" in this work. The paper is combinatorial in nature and focuses on counting and listing all possibilities in different classless of layouts. There has also been an attempt to explain a workflow for modeling architecture problems in graph theory. The authors of [7] identify several types of graphs useful in architecture. In their scheme the graphs of museum floor plans we consider are "Type IV".

There is limited past work on finding Hamiltonian walks in graphs of floor plans. The idea has been used as exercise in graph theory textbooks (see [8, Fig. 6.7] and [9, Fig 5.25]) and using Hamiltonian cycles to plan museum visits has been done at least twice. The authors of [10] devise a way to use crowd-sourced data to give recommendations for business travelers to make the most of a trip, which includes modeling museum floor plans as graphs. In [11], the authors describe a rigorous way to encode museum buildings as graphs, incorporating travel times and room sizes to be able to make estimates about how long it takes to walk through a museum. Our museum model is not as robust, since we ignore actual distance and only focus on the physical structure of the museum. However, we strive to clearly explain the mathematical motivation behind this idea and apply our framework to actual museum layouts.

Other uses of mathematics in museum design include the "art gallery problem" in computational geometry. This problem asks how many guards are necessary to watch a room at an art gallery. The sight lines of guards give way to polygons, which model the scenario. This problem has been explored in the context of the new MOMA building, using a "visibility graphs" [12].

1.3 Overview

The paper proceeds as follows. Section 2 provides a mathematical description of graph theory. Section 3 describes how we apply graph theory to model museum floor plans. Section 4 interprets several graphs from museum floor plans and Section 5 discusses implications and future work.

2 Graph Theory Primer

In graph theory, the word "graph" does not mean a graphical display of data or a "plot". Instead, a graph is a representation of connection. A graph consists of a set of objects called *vertices* and a set of *edges* that connect vertices. Vertices represent might represent computers, cities or people and edges represent connections between these objects, such as wires, highways or friendship status. In computer science applications, graphs are often called *networks*. An example of a graph is shown below.

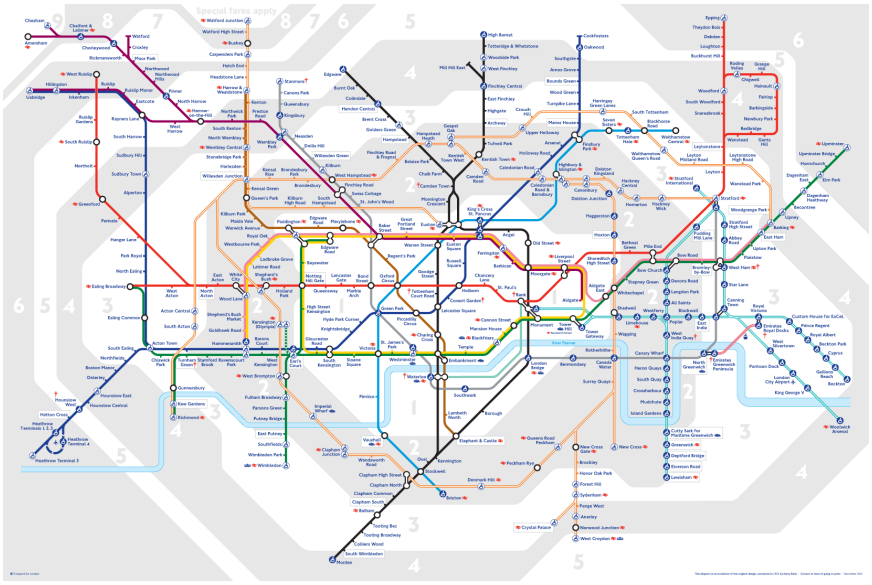


Figure 1: The London Underground Map is a graph. Vertices represent stations and edges represent subway lines. Notice that the stations on the map are not drawn in their actual relative locations, meaning the map is *topological*, rather than topographical. (Image: TfL)

What follows are key terms and examples that we will use in our analysis of museum floor plans. For a more robust introduction to graph theory, consult [8].

2.1 Graphs

Mathematical parlance mandates that we define precisely what we mean by the terms we use. We do this by writing definitions that relate new concepts to established ones. We start by defining a graph.[1]

Definition 2.1. A **graph** G consists of a set of vertices $V = \{v_1, v_2, \ldots, v_n\}$ and a set of edges $E = \{e_1, e_2, \ldots, e_m\}$. Each edge connects exactly two vertices. For each edge e in E we write $e = u \text{ — } v$, where u and v are vertices in V. We denote a graph by $G = (V, E)$.

Graphs are usually drawn by representing vertices as points and edges as line segments connecting pairs points. The example below shows how a graph is created from its vertex and edge sets.

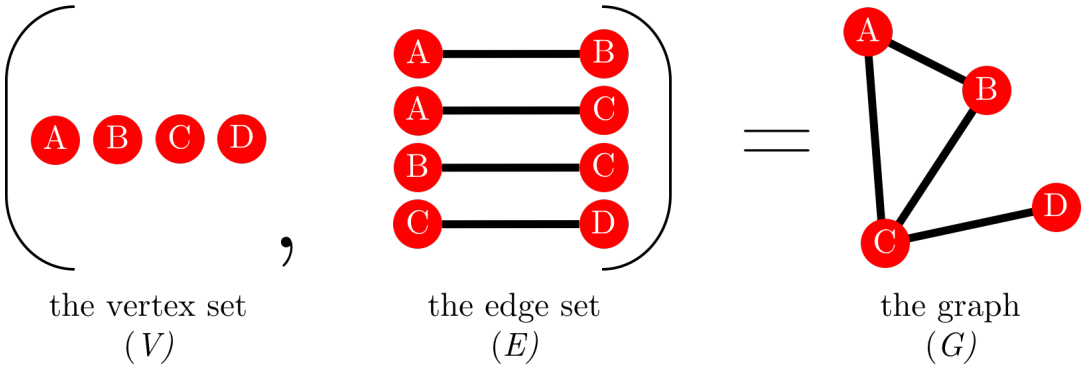


Figure 2: A visual representation of a graph with four vertices and four edges

For a graph $G = (V, E)$, the notation $|V|$ denotes the number of vertices in the graph and $|E|$ denotes the number of edges in the graph and so $|V| = 4$ and $|E| = 4$ in Figure 2.

The positions of the vertices in Figure 2 are arbitrary. Since it is useful to draw graphs in different shapes (with circular or branching structures, for instance), the field of *graph drawing* develops algorithms for visualizing graphs for different applications.

2.2 Walks

We are interested in identifying ways to move through museums by building floor plans as graphs. Appropriately, we use a concept in graph theory called a *walk* to traverse the graph.

Definition 2.2. Given a graph $G = (V, E)$, a **walk** is a sequence of distinct (different) vertices in V, in which successive pairs are connected by edges in E. The length of a walk is the number of edges in the walk.

Examples of walks in the graph in Figure 2 include

$$A \text{ — } B, \; A \text{ — } B \text{ — } C \text{ — } D, \text{ and } B \text{ — } A \text{ — } C \text{ — } B.$$

[1] Definitions for terms in graph theory vary between sources. Our construction of graph theory is mostly based on definitions given by Wolfram MathWorld (`mathworld.wolfram.com`).

The first walk has length 1 while the others have length 3. Note that a given walk does not necessarily contain all vertices or all edges in the graph and may repeat some vertices and edges.

The fact that the vertices in our graphs will represent rooms motivates the special type of walk defined below.

Definition 2.3. Given a graph $G = (V, E)$, a **Hamiltonian walk** is a walk that visits each vertex in V, starts and ends at the same vertex and has the shortest possible length. If each vertex is visited exactly once in a walk, the walk is called a **Hamiltonian cycle** and the graph G is called **Hamiltonian**.

Figure 3 shows two graphs: one is Hamiltonian and the other is not.



Graph 1 (G_1) Graph 2 (G_2)

Figure 3: It is impossible to find a Hamiltonian walk in Graph 1 without crossing the edge C — D twice and so Graph 1 is *not* Hamiltonian. Graph 2 has a Hamiltonian cycle (A — B — D — C — A) and so Graph 2 is Hamiltonian.

Finding a Hamiltonian cycle, which visits each vertex exactly once, will soon be equivalent to walking through all the rooms in a museum without repeating any rooms. Notice that the length of a Hamiltonian cycle will always be equal to the number of vertices in G. Hence, to find how close a graph is to possessing such a cycle we compare the length of the Hamiltonian walk to the number of vertices.

Definition 2.4. Given a graph $G = (V, E)$, the length of the shortest Hamiltonian walk minus the number of vertices in G, is known as the **Hamiltonian number**, denoted by $h(G)$.

Consider the graphs shown in Figure 3. Each graph has four vertices. A Hamiltonian walk through Graph 1 has length 5 and so $h(G_1) = 5 - 4 = 1$. (Even though Graph 1 is not Hamiltonian, it still has a Hamiltonian walk, which visits every vertex.) Graph 2 has a length 4 Hamiltonian cycle and so $h(G_2) = 4 - 4 = 0$. This fact about Graph 2 motivates the following *theorem*, or statement of truth.

Theorem 2.1. *Given a graph $G = (V, E)$, the Hamiltonian number of G is zero if and only if G is Hamiltonian.*

Hence, the Hamiltonian number is a way to measure how far a graph is from being Hamiltonian.

2.3 Connectivity

The *connectivity* of a graph refers the ease of traveling between vertices. There are many measure of connectivity, including diameter, average path length and density. To understand the concept of diameter, note that the *distance* between two vertices in a graph is defined as the length of the shortest walk between the two vertices. For example, there are many walks between A and B in Graph 1 in Figure 2, including A — B and A — C — B. The shortest length of a walk between A and B has length 1 and so the distance between A and B is said to be 1. The diameter of a graph is the longest shortest path, meant to intuitively represent the distance from one side to the other.

Definition 2.5. Given a graph $G = (V, E)$, the **diameter** of G is the longest distance between two vertices in V. The diameter of G is denoted by $\text{diam}(G)$. Additionally, the **average path length** of a graph is the average distance over all pairs of vertices. We denote the average path length of G by $\mu(G)$.

The diameter of the graph in Figure 2 is 2 since the longest distance between any two vertices is 2.

The density of a graph is a simply a function of the number of edges and vertices which describes how many edges a graph has accounting for its size. If a graph is completely connected (there is an edge between any pair of vertices), then density is 1. If a graph is completely disconnected (there are no edges) then density is 0.

Definition 2.6. The **density** of a graph $G = (V, E)$ is

$$D(G) = \frac{2|E|}{|V|(|V| - 1)}.$$

The density of the graph in Figure 2 is 0.75.

2.4 Case Study: The Bridges of Königsberg

In this paper we apply graph theory to a discussion about navigating rooms at museums. Interestingly, graph theory itself was created to solve a problem about touring the city of Königsberg, Prussia (now Kaliningrad, Russia).[2] The city was situated on the Pregel River and seven bridges connected each side of the city with two large islands. The "Königsberg Bridge Problem" asks: is it possible to walk around the city by crossing each bridge exactly once?

Legendary Swiss mathematician Leonhard Euler solved the problem by representing the different regions of the city as vertices and the bridges as edges of a graph, shown below.

[2]In fact, some math historians also identify this problem as a progenitor to the field of topology, which concerns itself with the shape and structure of objects rather than their actual positions.

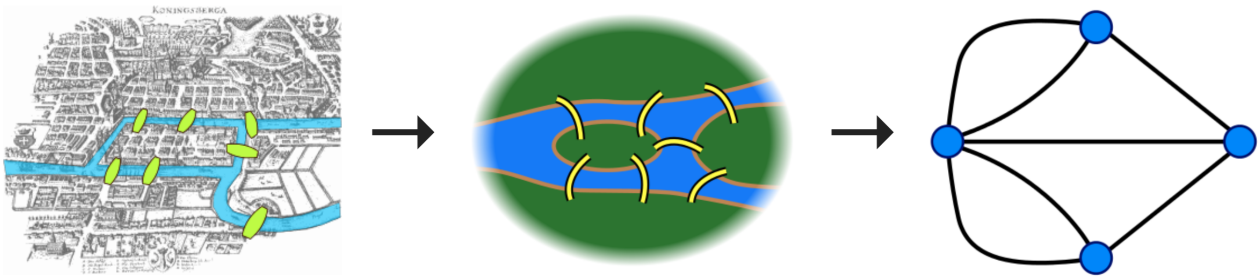


Figure 4: An illustration of the Königsberg Bridge Problem. The city map is summarized by the bridges connecting each region of land. This gives way to a graph representation of the problem. (Image: Wikimedia)

Euler proved that it is not possible to make such a walk and so, in his honor, a walk through a graph that visits every edge exactly once is now called an *Eulerian walk*. Note that Eulerian walks visit every edge exactly once and Hamiltonian walks visit every vertex exactly once.

3 Modeling Museum Maps

Armed with an introductory knowledge of graph theory, we apply the notion of a walk to study museum traversal, the way in which a person moves from room to room in a museum. To do this we represent museum floor plans as graphs. Simply put, vertices represent rooms and edges represent doorways.[3]

Definition 3.1. Given a floor plan of a building, construct a **floor plan graph** $G = (V, E)$ according to the following rules:

- The vertex set V is the set of rooms with exhibits.
- There is an edge between two rooms in E if and only if there is a doorway connecting the two rooms.

For simplicity we call the graph generated by a museum floor plan a **museum graph**.

Figure 5 is a museum graph generated by the floor plan for a portion of the National Gallery in London. Notice that this graph is an abstraction of the layout of the museum rooms, but it does not encode any information about the physical size and locations of the rooms. However, it does generate information about the shapes of routes through the museum. Immediately we notice a honeycomb-like structure of pentagons and hexagons, which is not obvious from the floor plan.

[3]Mathematically, this graph is the known as the *dual graph* of the floor plan itself.

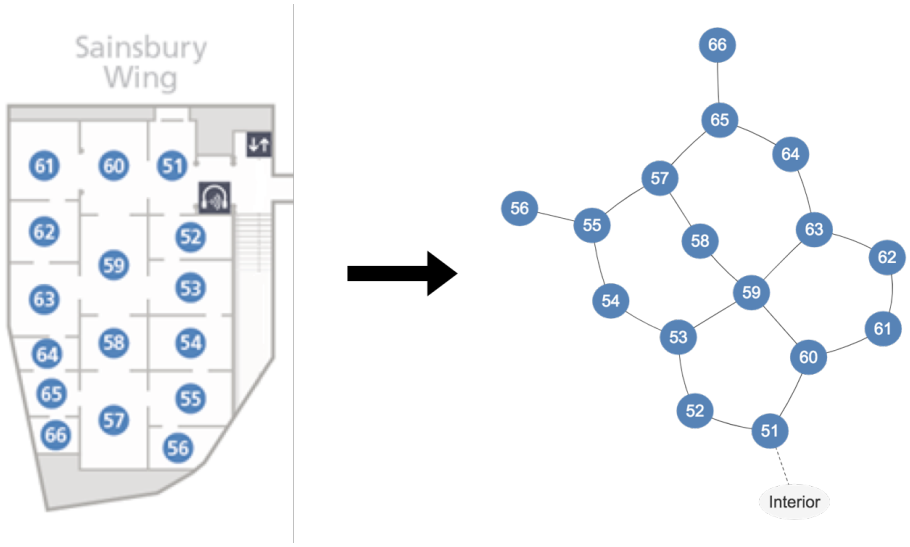


Figure 5: The floor plan for the Sainsbury Wing at the National Gallery represented as a floor plan graph.

3.1 Assumptions

Definition 3.1 describes how to construct a museum graph. Depending on the museum, we represent a floor plan in practice might be different. The assumptions below describe some of the changes that are made to accommodate real museum floor plans.

First, we assume each numbered room on a museum map is self-sufficient. "Sub-rooms" such as "34a" and "34b" are treated as one room if they are adjacent on the floor plan. In reality there may also be divisions inside rooms not shown on the map, but we assume that a visitor could make reasonable circuit around these rooms without retracing her steps.

Next, ambient hallways, stairs and escalators are all represented as one vertex if they are connected. This space includes the entry and exit ways, elevators, escalators and other neutral spaces that are not exhibition spaces. As a result, this vertex, which we call "interior", acts as the center of a museum graph, connecting the various wings of the building. A graph may have more than one interior vertex if there are multiple area like this that are not directly connected.

Similarly, the outside of the building is represented by the "exterior" vertex and it is only connected to the "interior" vertex. This vertex is included for technical reasons.[4] Any non-exhibit rooms such as bathrooms, shops and cafés are ignored. For convenience, some short hallways are represented as edges instead of connecting to the "interior" vertex.

3.2 Hamiltonian Walks in Museums

The graph measures introduced in Sections 2.2 and 2.3 allow us to compare and analyze the structure of museum layouts. Recall the Hamiltonian number introduced in Definition 2.4. This is the number of "extra" edges required to visit every vertex in a graph in the shortest

[4]In particular, the algorithm we use to search for walks requires that the starting and ending vertices be different.

possible walk starting and ending at the same vertex. If a museum graph G has a low Hamiltonian number, then we can walk through the museum and visit every room and $h(G)$ is the number of extra room visits required. This number scales with the size of the graph (i.e. a large graph will inherently have a larger Hamiltonian number) and so we rescale it by the number of edges in the graph to avoid penalizing larger graphs.

Definition 3.2. Given a floor plan graph $G = (V, E)$, define the **adjusted Hamiltonian number** by

$$\tilde{h}(G) = \frac{h(G) - 1}{|V| - 1}.$$

The factor of one in the numerator and the denominator accounts for the "exterior" vertex, which is simply a proxy for returning to the "interior" of the building at the end of a walk. This new measure allows us to compare museums floor plan graphs by how close they come to being Hamiltonian. In physical terms this is how close the museum is to possessing a walk that visits every room in the museum exactly once.

As an example, consider the museum graph of the Sainsbury Wing in Figure 5. The following is a Hamiltonian walk through the Sainsbury Wing:

$$\begin{gathered} 51 \text{ — } 52 \text{ — } 53 \text{ — } 54 \text{ — } 55 \text{ — } 56 \sim 66 \sim 64 \text{ — } 65 \text{ — } 57 \\ \text{— } 58 \text{ — } 59 \text{ — } 63 \text{ — } 62 \text{ — } 61 \text{ — } 60 \sim \text{Interior} \end{gathered}$$

The $\sim$ symbol indicates that this part of the walk contains more than one edge. In this case $56 \sim 66$ has distance 4; $66 \sim 64$ has distance 2; and $60 \sim$ "Interior" has distance 2. This means that at least 4 extra room visits are required to visit every room and the Hamiltonian number is 4. Since the graph as 17 vertices, the adjusted Hamiltonian number is 0.25.

Recall that our museum floor plan graphs include an "interior" vertex. Even if two vertices are nearby, the presence of this vertex adds an extra edge to a walk if it is necessary to pass through the interior of the building (taking stairs, elevators or walking through a non-exhibit space). Thus, an algorithm searching for a Hamiltonian walk is disincentived from stepping outside of exhibit areas, even if it is physically quicker than moving between rooms.

3.3 Connectivity

Connectivity measures tell us something about the relative ease of moving about the museum. In the context of a museum graph, diameter is the maximum number of doorways required to move from one room in the museum to any other room and the average path length is the average number of doorways someone will walk through to pass between any pair of rooms. The diameter of the Sainsbury Wing graph is 7 since the longest distance between a given pair of vertices ("Interior" $\sim$ 66) is 7. The average distance is 3.014.

On the other hand, density indicates how many doorways between rooms exist versus how many could exist. The Sainsbury Wing graph has 17 vertices and 21 edges and so, using Definition 2.6, density is 0.154.

3.4 Computational Notes

In complexity theory, the problem of finding Hamiltonian walks is said to be NP-Hard. Theoretically speaking, there are no known solutions with reasonable runtime for problems that are NP-Hard. This mean that for large graphs, finding Hamiltonian walks is not feasible because the computation takes too long.

For our analysis we reduce the problem of finding Hamiltonian walks to solving the "Traveling Salesman Problem" (TSP). The TSP is motivated by a salesman who wishes to visit a series of cities in the most efficient way possible without repeating cities and returning home at the end. To solve the Hamiltonian problem, we compute the distances between vertices in the graph (by finding shortest paths) and feed this information to an algorithm that gives an approximate solution to the TSP. [5]. For a graph-theortic study of the London Underground using the TSP, see [13].

4 Analysis

We compute the graph measures described in Section 3 on museum graphs created from actual museum floor plans of three London museums: the National Portrait Gallery, the National Gallery and the British Museum.

None of the museum graphs we consider are truly Hamiltonian and so it is not possible to visit every room in each museum without repeating some rooms. The amount of repeat necessary is captured by the Hamiltonian number, $h(G)$, which can be made comparable to other graphs using the adjusted Hamiltonian number, $\tilde{h}(G)$). The diameter of the graph, $\text{diam}(G)$, the mean distance, $\mu(G)$ and the density, $D(G)$ are measures of how connected the museum is and how easy it is to move between different rooms.

Since it can be difficult to convey all of the structure information in a static graphic, we created interactive versions of all of the museum graphs in this paper. To access, visit egr.msu.edu/∼eithunmi/london/museums.

4.1 Summary

Details about the structure of each museum floor plan graph are collected in Table 1.

Museum	$\lvert V\rvert$	$\lvert E\rvert$	$h(G)$	$\tilde{h}(G)$	$\mu(G)$	$\text{diam}(G)$	$D(G)$
National Portrait Gallery	44	56	19	0.44	3.76	8	0.059
National Gallery	72	92	27	0.38	5.72	14	0.036
British Museum (South to North)	70	86	31	0.45	4.94	12	0.036

Table 1: Heuristic comparison of museum graphs

4.2 The National Portrait Gallery

The National Portrait Gallery has 44 rooms connected by 46 doorways. The floor plan is shown in Figure 6.

[5] We use the tsp-solver module in Python 3.

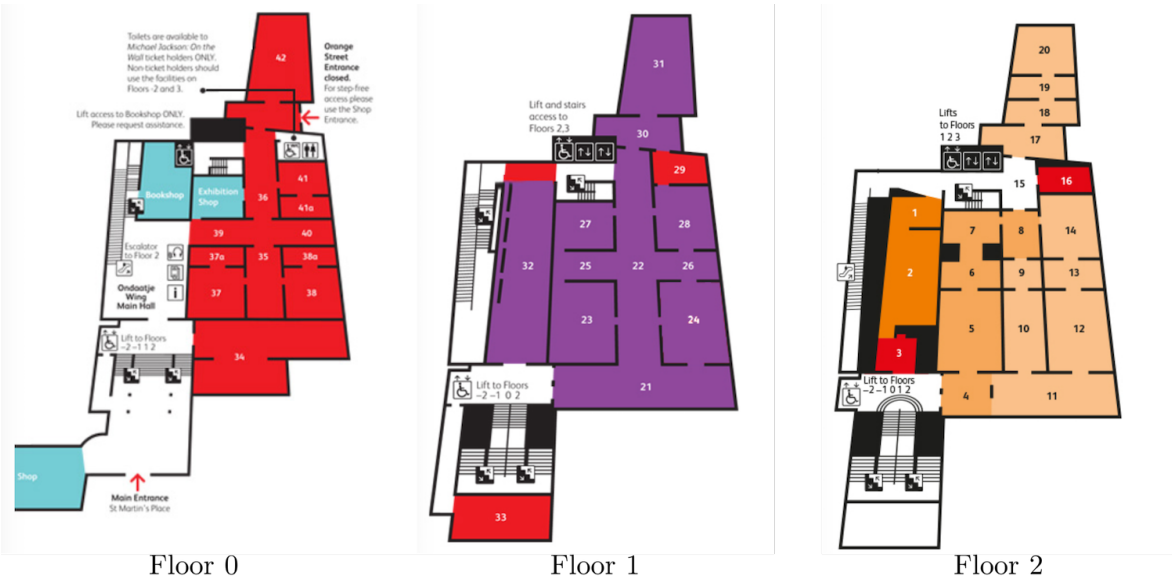


Figure 6: Floor plan for the National Portrait Gallery.

Notice that there are distinct wings of the museum denoted by different colors: "red" indicates a special exhibition, "purple" indicates a Victorian era exhibit, and so on. The wings of the museum correspond to clusters of vertices in the museum graph, shown in Figure 7.

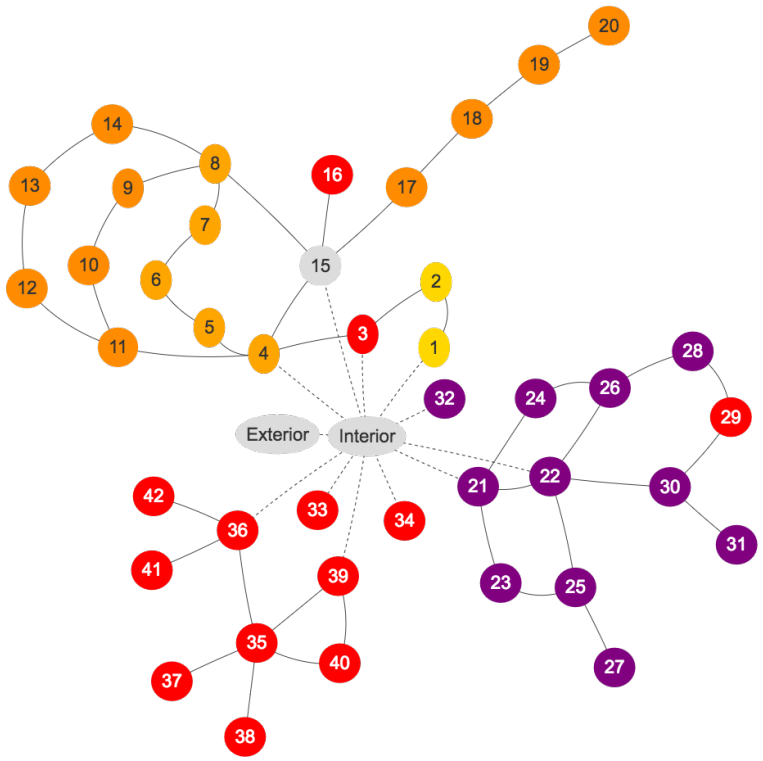


Figure 7: Museum graph for the National Portrait Gallery

Referring to Table 1, the graph has a relatively high density, meaning that many doorways exist between rooms. The diameter of the graph is 8 since one must cross through 8 doorways to walk from Room 20 to Rooms 27, 28, 29, 31, 37 and 38. Notice that Room 20 is only reachable by passing through rooms 15, 17, 18 and 19, creating a linear structure in this area of the museum.

The Hamiltonian number $h(G) = 19$ indicates that there are 19 room visits necessary beyond visiting every room once to make a Hamiltonian walk. These extra visit are recounted in Table 2 for the Hamiltonian walk found by our algorithm.

Room	**Extra Visits**	**Room Name**
Interior	5	Hallways
8	1	The Later Stuarts
15	2	Edward William Lane
17	1	Royalty, Celebrity and Scandal
18	1	Art, Invention and Thought: The Romantics
19	1	Art, Invention and Thought: Making the Modern World
22	1	Statesmen's Gallery
25	2	Women, Politics and Domestic Life in Victorian Britain
30	1	Britain 1901-1914
35	2	Special Displays
36	2	Special Displays

Table 2: Rooms visited more than once on our Hamiltonian walk through the National Portrait Gallery.

The Interior vertex is visited many times to move between the wings of the museum and rooms 17, 18, 19 are visited more than once because this is necessary to get to Room 20 and back. The other repeat visits are necessitated by the structure of the museum layout.[6] This type of analysis has implications for where museums decide to place objects: rooms along walks through the museum that are visited multiple times will be seen more often.

4.3 The National Gallery

The floor plan and museum graph for the National Gallery are shown in Figure 8.

[6]Hamiltonian walks are not necessarily unique and so an optimal walk through the museum does not necessarily visit these exact vertices, but they are indicative of the kind of overlap necessary to make an optimal walk through every room.

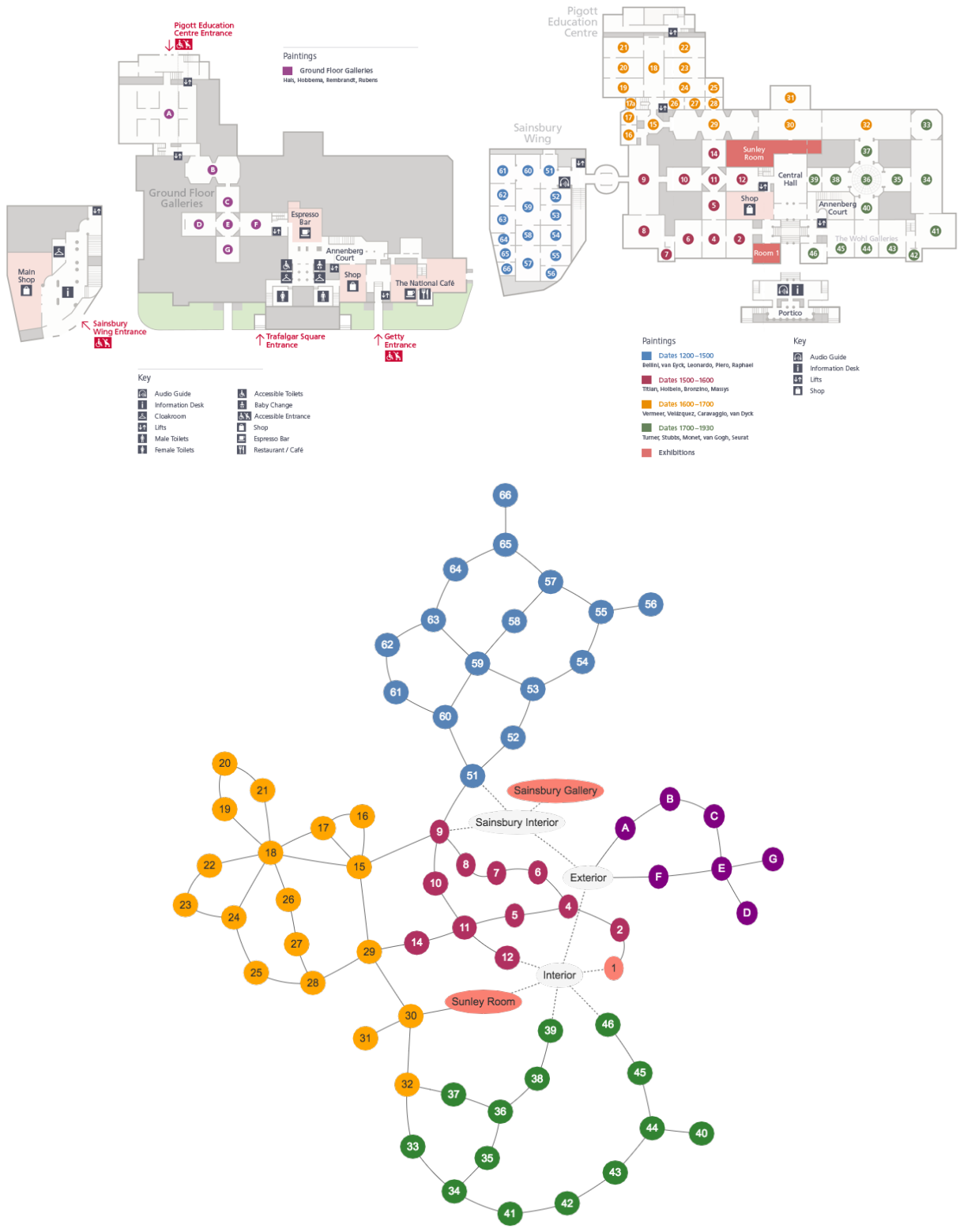


Figure 8: Above: Floor plan for the National Gallery for floors 0 and 2. Below: Museum graph.

The National Gallery is the expansive of museums we are considering, with 72 rooms and 92 doorways. The diameter of the museum graph is 14, indicating that the farthest distance between two rooms is 14 doorways. This occurs traveling from Room 66 to Rooms 41 and 42. Compared to the other museums, the diameter is highest, the average distance between rooms is highest (5.72) and the density is lowest, which suggests that it is more difficult to move between random pairs of rooms in this museum than the others.

On the other hand, the adjusted Hamiltonian number for this graph is small. Hence, accounting for its size, this museum has the shortest walk that visits every room among the three museums. The National Gallery has two distinct entrances: the main entrance from Trafalgar Square and the entrance into the Sainsbury wing. Hence, we have two interior vertices labeled "Interior" and "Sainsubry Interior", and both are connected to the "Exterior". Interestingly, the Hamiltonian walk found by our algorithm (starting in the main "Interior") goes outside, traveling from Room 12 to Room A by going out from the main entrance and into the Sainsbury wing. Of course, this segment of the walk is likely inefficient in terms of actual time spent touring the museum, but it represents a possible best way to move about the museum from a structural standpoint.

4.4 The British Museum

The British Museum is known for its massive size. The collection contains 8 million artifacts and about 1% are on display [14].[7] However there are three more exhibit rooms at the National Gallery, indicating that two museums are actually comparable in size. The floor plan and museum graph for the British museum are shown in Figure 9.

The museum graph for the British museum has the highest adjusted Hamiltonian number and so the Hamiltonian walk found by our algorithm is relatively large, requiring 31 visits to previously visited rooms. However, with diameter and density measures close to those of the the National Gallery, the British Museum has a similar degree of connectivity. In terms of doorways, the two farthest rooms are 52 and 15, which have 11 intermediate rooms.

The visitor map of the British museum lists several "don't miss" objects and the rooms where they are located. By liming our search for a Hamiltonian walk to just these rooms, we can also find the shortest route that visits all of these exhibits and observe how many other rooms we visit on the way. This path is shown in Figure 10.

The suggested order, starting from the South Entrance, is 56, 70, 10, 18, 4, 40, 52, 2, 33, 25, 24. The only adjacent rooms are 24 and 25 and so there are numerous other rooms that are visited during the walk. The Hamiltonian number is 27, indicating that there are 27 "extra" room visits. There are also 27 rooms total are visited on the walk (including the north and south interiors), meaning that there 14 extra exhibit rooms that must be visited beyond the "don't miss" rooms.

[7] As a fascinating aside, the British Museum holds several Ancient icosahedral (20-sided) dice from Egypt that can be seen as Hamiltonian cycles. This was discovered by legendary computer scientist Donald Knuth in a fasicle for his book series *The Art of Computer Programming* [15].

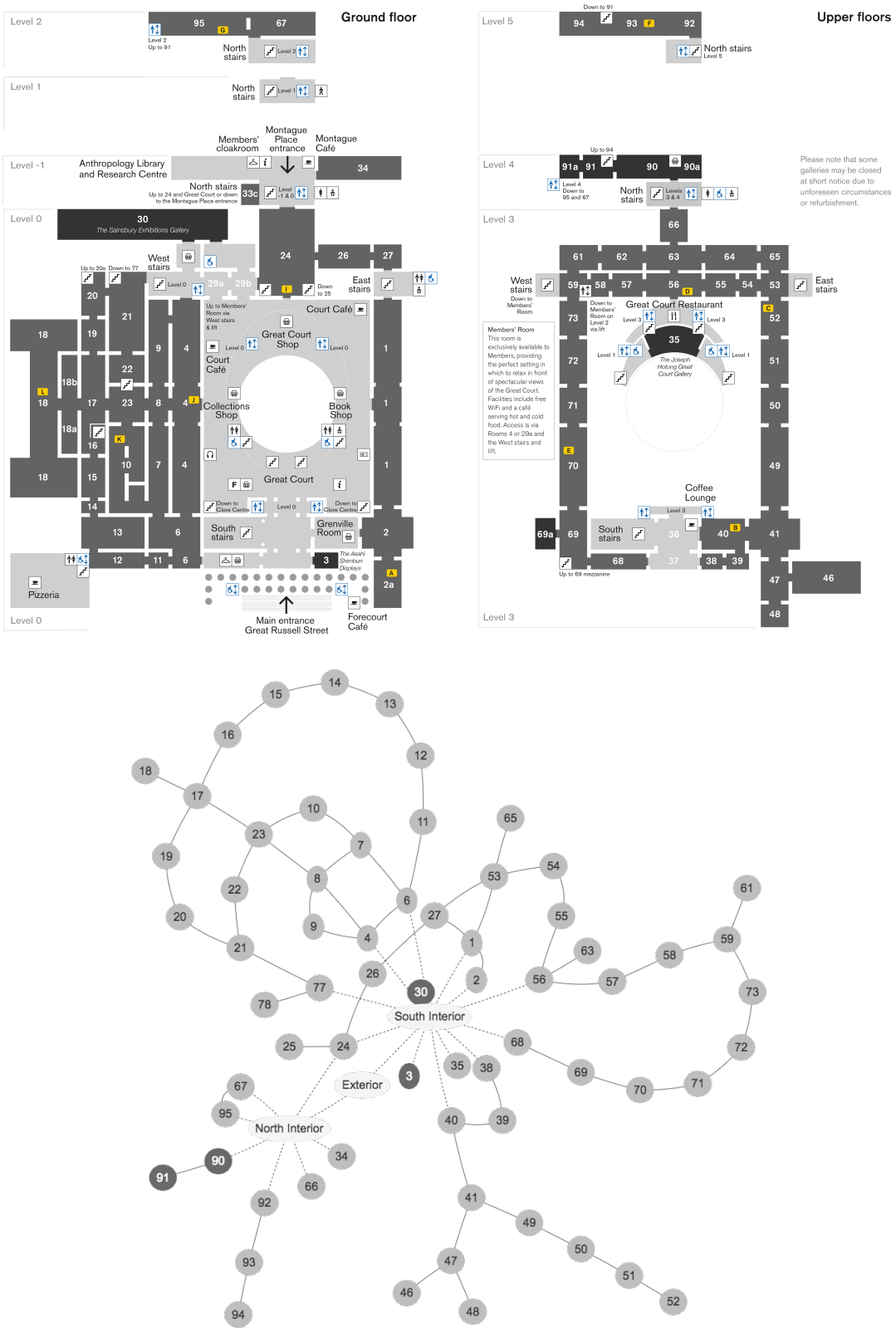


Figure 9: Above: Main floor plan for the British Museum. Below: Museum graph.

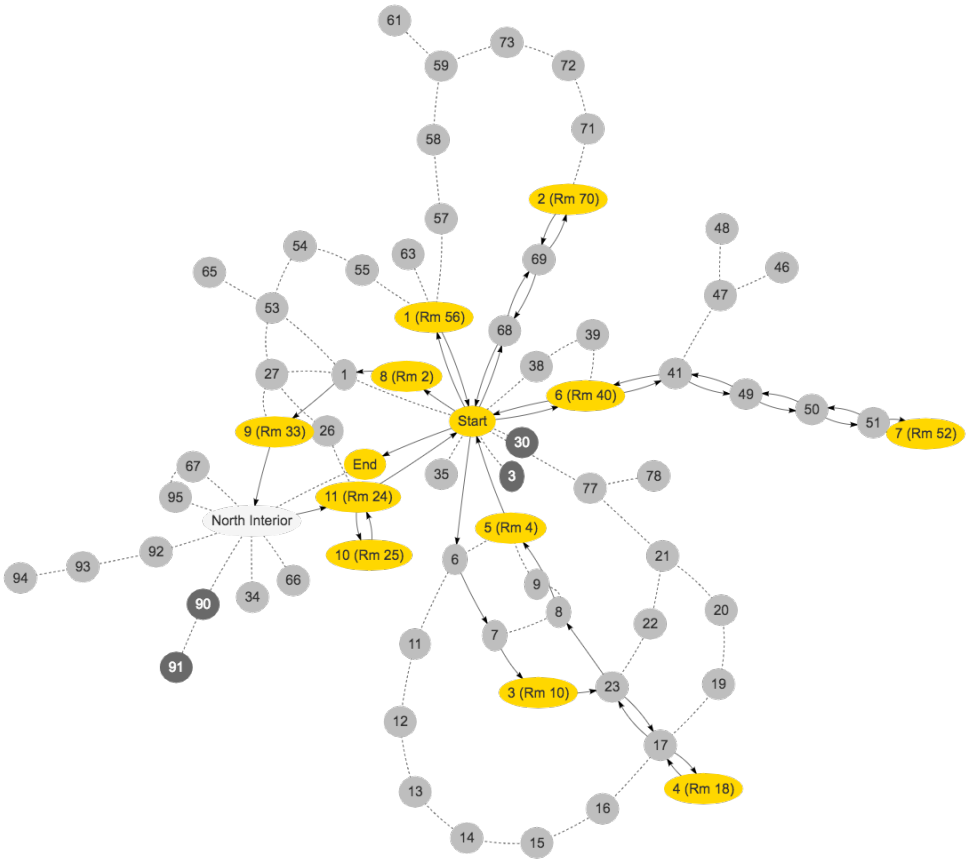


Figure 10: A Hamiltonian walk through the rooms with "don't miss" objects. The rooms are shown in gold and paths between rooms are given arrows.

5 Discussion

In this work we describe how a museum floor plan can be represented as a graph to study the structure and connectivity of a museum and find optimal walks covering a set of rooms. Modeling museum floor plans has graph as implications for building museums and galleries in a way that allows for shorter paths between points of interests. Museums, or wings of museums, that have a low Hamiltonian number, and thus a shorter Hamiltonian walk, allow visitors to walk through many rooms without visiting rooms more than once. However, an ideal floor plan may be difficult to build (or even define), and so knowing which rooms are likely to be visited more often should help curators decide where to place important exhibits.

The structure of the floor plan, given by a museum graph, provides considerations for what people see. There are two important questions in this discussion: "What do visitors want to see?" and "What do curators want visitors to see?" Without taking into account some of the particulars of the museum layout, museum-goers will be guided by their own whims and limited by how the physical building allows them to move. An important part of the process of exhibit placement is to collect data and make observations about how people use museums and meld them with curators' goals. For example, Beverly Serrell discovered

that visitors tend to turn right in museums and then move anti-clockwise [4]. Thus, a museum's rooms should be configured in a way that acknowledges common traffic patterns and facillitate clear routes to important objects.

A more concrete use of museum graphs is to create customizable museum guides. This is the goal in [10, 11]. Museums often provide audio guides to visitors to help interpret objects and displays. These guides could also include route planning options to allow visitors to rank which exhibits they would like to visit so they can be given a custom visit plan based on how much time they have to explore. For visitors unsure of what to see, highlight tours could be offered, a feature already on many museum maps (see Section 4.4 about British Museum). Currently the Louvre website facilitates route planing by offering "trails" through the museum [16]. The mathematics need to accomplish this task is graph theory and builds on the tools we have developed here.

Furthermore, route planning can also be a new way for museums to integrate technology into the visitor experience. With a massive push for museums to allow cell phones and incorporate more virtual and augmented reality, including digital route planning could be another to interact with visitors. The British Museum uses Facebook insights to track public interest in their exhibitions [17]. Using a custom museum guide, this data could be collected on a more personal level to help the museum learn about how visitors are using the space and meet their goal of using more technology.

On some level, it is reasonable to ask why these details matter. According to some museum experts, there an extreme amount of variability in how and how often people make use of their time at museums [4]. Another way to view museum graphs is as an artistic inspiration. The abstraction of the room layout ignores many details of the physical building, leaving important information about connections between rooms. Museums, particularly art museums, focus on providing ample space to view art and the layout of the museum itself should also reflect this goal. Analyzing the structure of museums could help curators reflect on the space that the museum offers and how it will help them achieve their goals in presenting the material. This leads to a broader metaphorical discussion of architectural goals, like the habit of medieval builders to design cathedrals in the shape of a cross.

With several long-term goals in mind, including making better use of museum space and designing a route planning guide, future work in this area includes building a larger and more detailed dataset of museum graphs. This would help explore questions about how to better represent museums as graphs and how to handle more arcane museum layouts. A more detailed specification of a museum graph would include travel times and the contents of each room (like in [11]). It would also be interesting to compare graphs to study the relative ease of moving museums and predominant features of museum layouts, such as the ring-like structures in Japanese museums [18]. This whole discussion should also consider accessibility to ensure that all museum guests are able to experience the museum to the fullest extent possible.

References

[1] Benjamin Sly. The british museum: the egyptian room, with visitors. engraving by radclyffe after b. sly, 1844. *Wellcome Collection*. wellcomecollection.org/works/fv77wmdq.

[2] Christina Goulding. The museum environment and the visitor experience. *European Journal of Marketing*, 34(3/4):261–278, April 2000.

[3] Morris Hargreavess McIntyre. Touching History: An evaluation of Hands On desks at The British Museum. page 47, July 2008.

[4] Laurent Carpentier. Inside the mind of the museum-goer. *The Guardian*, October 2015.

[5] Koenraad Cuypers, Steinar Krokstad, Turid Lingaas Holmen, Margunn Skjei Knudtsen, Lars Olov Bygren, and Jostein Holmen. Patterns of receptive and creative cultural activities and their association with perceived health, anxiety, depression and satisfaction with life among adults: the hunt study, norway. *Journal of Epidemiology & Community Health*, 66(8):698–703, 2012.

[6] Robin Wilson. *Applications of Graph Theory*. Academic Press, London, 1979.

[7] J. Roth and R. Hashimshony. Algorithms in graph theory and their use for solving problems in architectural design. *Computer-Aided Design*, 20(7):373–381, September 1988.

[8] Gary Chartrand and Ping Zhang. *A First Course in Graph Theory*. Dover Publications, May 2013. Google-Books-ID: zA_CAgAAQBAJ.

[9] Fred Buckley and Marty Lewinter. *Introductory Graph Theory with Applications*. Waveland Press, November 2013. Google-Books-ID: 54tIAgAAQBAJ.

[10] Thomas Collerton, Andrea Marrella, Massimo Mecella, and Tiziana Catarci. Route recommendations to business travelers exploiting crowd-sourced data. In Muhammad Younas, Irfan Awan, and Irena Holubova, editors, *Mobile Web and Intelligent Information Systems*, pages 3–17, Cham, 2017. Springer International Publishing.

[11] Daniel Le Berre, Pierre Marquis, and Stephanie Roussel. Planning Personalised Museum Visits. *Proceedings of the Twenty-Third International Conference on Automated Planning and Scheduling*, page 9, 2013.

[12] Ipek Kaynar Rohloff. Aligning Museum Building Projects with Institutional Goals: A Visitor Experience Centered Approach. *UM Working Papers in Museum Studies*, (6):30, 2011.

[13] M. Drozdowski, D. Kowalski, J. Mizgajski, D. Mokwa, and G. Pawlak. Mind the gap: A study of Tube tour. *Computers & Operations Research*, 39(11):2705 – 2714, 2012.

[14] The British Museum: Fact Sheet. britishmuseum.org/pdf/fact_sheet_bm_collection.pdf.

[15] Donald Knuth. *The Art of Computer Programming*, volume 4. Pre-fascicle 8A. www-cs-faculty.stanford.edu/∼knuth/fasc8a.ps.gz.

[16] Louvre: Visitor trails. louvre.fr/en/parcours.

[17] Anastasia-Yvoni Spiliopoulou, Simon Mahony, Vassilis Routsis, and Christina Kamposiori. Cultural institutions in the digital age: British Museums use of Facebook Insights. *Participations: Journal of Audience & Reception Studies*, 11(1):18, 2014.

[18] Joseph Wong. Conceptual engawa: The experience of ring-based circulation in Tadao Ando museums. *Environment and Planning B Planning and Design*, 41, January 2014.